

A KK-monopole giant graviton in $AdS_5 \times Y_5$

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ABSTRACT: We construct a new giant graviton solution in $AdS_5 \times Y_5$, with Y_5 a quasi-regular Sasaki-Einstein manifold, consisting on a Kaluza-Klein monopole wrapped around the Y_5 and with its Taub-NUT direction in AdS_5 . We find that this configuration has minimal energy when put in the centre of AdS_5 , where it behaves as a massless particle. When we take Y_5 to be S^5 , we provide a microscopical description in terms of multiple gravitational waves expanding into the fuzzy S^5 defined as an S^1 bundle over the fuzzy CP^2 . Finally we provide a possible field theory dual interpretation of the construction.

KEYWORDS: AdS-CFT Correspondence, p-branes.

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1. Introduction

As it is well-known, giant gravitons are stable brane configurations with non-zero angular momentum, that are wrapped on $(n - 2)$ - or $(m - 2)$ -spheres in $AdS_m \times S^n$ spacetimes and carry a dipole moment with respect to the background gauge potential [1–4]. They are not topologically stable, but are at dynamical equilibrium because the contraction due to the tension of the brane is precisely cancelled by the expansion due to the coupling of the angular momentum to the background flux field. These spherical brane configurations turn out to be massless, conserve the same number of supersymmetries and carry the same quantum numbers of a graviton.

Giant graviton configurations were first proposed as a way to satisfy the stringy exclusion principle implied by the AdS/CFT correspondence [1]. The spherical $(n - 2)$ -brane expands into the S^n part of the geometry with a radius proportional to its angular momentum. Since this radius is bounded by the radius of the S^n , the configuration has associated a maximum angular momentum. The $(m - 2)$ -brane giant graviton configurations [2, 3], on the other hand, expand into the AdS_m part of the geometry, and they do not satisfy the stringy exclusion principle. For a discussion on the degeneracy of these two types of giant gravitons and the point-like graviton, we refer for instance to [5] and references therein.

The construction of giant gravitons has also been generalised to $AdS_5 \times Y_5$ spacetimes, where Y_5 is a Sasaki-Einstein manifold. In [6] and [7] a D3-brane wrapped around the angular S^3 of the AdS_5 and moving along the Reeb vector of the Sasaki-Einstein space was considered, yielding a dual giant graviton. A generalisation of giant graviton configurations preserving 1/4 or 1/8 of the supersymmetries has also been considered by Mikhailov [8] in the $AdS_5 \times T^{1,1}$ spacetime.

In this paper we find a new giant graviton configuration in $AdS_5 \times Y_5$, which consists of a Kaluza-Klein (KK) monopole with internal angular momentum, wrapping the Y_5 part of the geometry and with Taub-NUT direction in the AdS_5 part. This solution has distinguishing features with respect to the previous giant graviton solutions constructed in the literature. First, the monopole does not couple to the 4-form potential of the background and the configuration is therefore not at a dynamical equilibrium position. Still it is stable because by construction it is wrapped around the entire Y_5 . Secondly, it has a fixed size L , the “radius” of the Y_5 , independent of the momentum of the configuration. In fact the energy of the monopole depends only on its position in the AdS_5 part of the spacetime, and it is minimised when the monopole sits at the centre of AdS_5 , where it behaves as a massless particle. In this sense this new giant graviton configuration does not provide a realisation of the stringy exclusion principle. However, its mere existence is sufficiently surprising to motivate a closer look at the configuration. Furthermore, the fact that the giant graviton is built up from a Kaluza-Klein monopole, could lead to interesting view points in the context of the AdS/CFT correspondence.

The organisation of this paper is as follows. In section 2 we present the Kaluza-Klein monopole giant graviton solution. We start by introducing our probe monopole and then construct an action suitable to describe it. We then calculate the energy of the configuration and show that when the monopole sits at the centre of AdS_5 it behaves as a massless particle. In section 3 we move to consider the microscopical description of this configuration in terms of expanding gravitational waves. Given that the fuzzy version of an arbitrary Sasaki-Einstein manifold is not known, we particularise to the case in which $Y_5 = S^5$. The fuzzy 5-sphere that we consider is defined as an S^1 bundle over the fuzzy CP^2 . This fuzzy manifold has been successfully used in the microscopical description of 5-sphere giant gravitons [9, 10], and of the baryon vertex with magnetic flux [11]. In these examples the fibre structure of the S^5 plays a crucial role in the construction. Finally in section 4 we present a candidate description of our configuration in the field theory side. We end with some conclusions in section 5.

2. A new giant graviton solution

2.1 The Kaluza-Klein monopole probe

Consider the $AdS_5 \times Y_5$ spacetime, with Y_5 a quasi-regular five-dimensional Sasaki-Einstein manifold. All these Sasaki-Einstein manifolds have a constant norm Killing vector, called the Reeb vector. For the cases we are interested in, the $U(1)$ action of the Reeb vector is free and the quotient space is (at least locally) a four-dimensional regular Kähler-Einstein manifold M_4 with positive curvature. In that case the metric on Y_5 can (at least locally) be written as a $U(1)$ fibre bundle over the M_4 ,

$$ds_Y^2 = ds_M^2 + (d\psi + B)^2, \tag{2.1}$$

where ds_M^2 is the metric on the M_4 and the Killing vector $k^\mu = \delta_\psi^\mu$ is the Reeb vector. The Kähler form on M_4 is related to the fibre connection B via $\omega_M = \frac{1}{2}dB$.¹ The $AdS_5 \times Y_5$

¹For an extensive summary on the properties of Sasaki-Einstein manifolds we refer to [12].

background contains as well a non-vanishing 4-form RR-potential.

Using the U(1) decomposition above, the metric of $AdS_5 \times Y_5$ can be written as

$$ds^2 = -\left(1 + \frac{r^2}{L^2}\right)dt^2 + \frac{dr^2}{\left(1 + \frac{r^2}{L^2}\right)} + \frac{r^2}{4} \left[d\Omega_2^2 + (d\chi + A)^2 \right] + L^2 \left[ds_M^2 + (d\psi + B)^2 \right], \quad (2.2)$$

where we have used global coordinates in the AdS part and written the angular S^3 contained in AdS_5 as a U(1) fibre over S^2 . A and B stand for the connections of the S^3 and Y_5 fibre bundles respectively. In these coordinates, the fibre directions χ and ψ are clearly globally defined isometry directions.

Consider now a KK-monopole wrapped on the Y_5 , with Taub-NUT direction χ and propagating along ψ . This will be our KK-monopole probe. In order to study the dynamics of this monopole we start by constructing an action suitable to describe it.

The effective action describing the dynamics of the Type IIB Kaluza-Klein monopole was constructed in [13]. Like the Type IIA NS5-brane, to which it is related by T-duality along the Taub-NUT direction, the action for the monopole is described by a six-dimensional (2,0) tensor supermultiplet, which contains a self-dual 2-form \hat{W}_{ab}^+ and 5 scalars $\{X^i, \omega, \tilde{\omega}\}$. The self-dual 2-form is associated to the (S-duality invariant) configuration of the monopole intersecting a D3-brane, wrapped on the Taub-NUT direction. The worldvolume scalars ω and $\tilde{\omega}$ are associated with the intersections of D5- and NS5-branes respectively, and form a doublet under S-duality. Finally the scalars X^i (with $i = 1, 2, 3$) are the embedding scalars, that describe the position of the monopole in the transverse space. Note that although the worldvolume of the monopole is six-dimensional, its position is specified by only three embedding scalars. This is because the Taub-NUT direction is considered to be transverse, but being an isometry direction it does not yield a dynamical degree of freedom. The KK-monopole action takes in fact the form of a gauged sigma model, where the degree of freedom corresponding to the Taub-NUT direction is gauged away [14]. Due to the presence of the self-dual two-form \hat{W}_{ab}^+ , there is no straightforward covariant formulation of the action (see for example [15]). However, like in the case of the five-brane [16, 17], it is possible to give an approximation, expanding the action to quadratic order in the self-dual two-form.

In our case, the situation is actually simpler. As our KK-monopole probe is wrapped around Y_5 , the U(1) fibre direction ψ is contained in its worldvolume. Therefore it is possible to effectively compactify the monopole over the fibre direction and to consider the (much simpler) action for a wrapped KK-monopole. Moreover, momentum charge along this U(1) fibre direction can easily be induced by switching on an appropriate magnetic flux in the worldvolume.

The field content of the wrapped monopole is given by the five-dimensional (1,1) vector multiplet, which contains 5 scalars and one vector, and is the dimensional reduction of the six-dimensional (2,0) tensor multiplet. The self-duality condition becomes a Hodge-duality condition between the vector and a two-form, which does not appear explicitly in the action.² In this way an action can be constructed to all orders in the field strength. In

²This is very similar to the M5-brane case. The unwrapped M5-brane contains a self-dual 2-form

practise, the action of the wrapped monopole is most easily constructed from the action of the Type IIA KK-monopole, as the latter has the six-dimensional (1, 1) vector multiplet as its worldvolume field content [18]. After T-dualising along a worldvolume direction, the resulting action describes a Type IIB monopole wrapped along the T-duality direction, with an effectively five-dimensional worldvolume. As our KK-monopole probe is wrapped on the S^1 fibre direction of the Y_5 , its spatial worldvolume becomes effectively $\mathbb{R} \times M_4$.

2.2 The action for the wrapped monopole

The starting point is the action for the Type IIA Kaluza-Klein monopole constructed in [18], which we compactify along a worldvolume direction and T-dualize. The resulting action describes a Type IIB Kaluza-Klein monopole which is wrapped on the T-duality direction and has, effectively, a five-dimensional worldvolume. The T-duality direction appears in the action as a new isometric direction, whose Killing vector we denote by k^μ . On the other hand the Killing vector associated with the Taub-NUT direction is denoted by ℓ^μ . The explicit action is given by

$$S = -T_4 \int d^5\sigma e^{-2\phi} k \ell^2 \sqrt{|\det(D_a X^\mu D_b X^\nu g_{\mu\nu} + e^\phi k^{-1} \ell^{-1} \mathcal{F}_{ab})|} - T_4 \int d^5\sigma \left\{ P[i_k i_\ell N^{(7)}] - P[i_\ell C^{(4)}] \wedge \mathcal{F} - \frac{1}{2} P\left[\frac{k^{(1)}}{k^2}\right] \wedge \mathcal{F} \wedge \mathcal{F} + \dots \right\}, \quad (2.3)$$

where the scalars k and ℓ are the norm of k^μ and ℓ^μ respectively, $k^{(1)}$ denotes the 1-form with components k_μ and $(i_\ell i_k \Omega)_{\mu_1 \dots \mu_n} = \ell^\rho k^\nu \Omega_{\nu \rho \mu_1 \dots \mu_n}$. In this action the pull-backs into the worldvolume are taken with gauge covariant derivatives

$$D_a X^\mu = \partial_a X^\mu - k^{-2} k_\nu \partial_a X^\nu k^\mu - \ell^{-2} \ell_\nu \partial_a X^\nu \ell^\mu, \quad (2.4)$$

which ensure local invariance under the isometric transformations generated by the two Killing vectors

$$\delta X^\mu = \Lambda^{(1)}(\sigma) k^\mu + \Lambda^{(2)}(\sigma) \ell^\mu. \quad (2.5)$$

In this way the embedding scalars corresponding to the isometry directions are eliminated as dynamical degrees of freedom and the action is given by a gauged sigma model of the type first considered in [14].

The two-form field strength \mathcal{F} is defined as

$$\mathcal{F} = 2\partial V^{(1)} + P[i_k i_\ell C^{(4)}], \quad (2.6)$$

where the worldvolume vector field $V^{(1)}$ is the T-dual of the vector field of the Type IIA monopole (or, alternatively, the dimensional reduction of the self-dual two-form \hat{W}^+). While in the Type IIA monopole the vector field is associated to D2-branes wrapped on the Taub-NUT direction, in the IIB case it is associated to D3-branes, wrapped on both Killing directions.

in its worldvolume, whereas the M5-brane wrapped on the eleventh direction (the D4) depends on an unconstrained five-dimensional vector field.

The action of the Type IIA monopole contains as well a worldvolume scalar associated to strings wrapped on the Taub-NUT direction. This field gives, upon T-duality, a world-volume scalar ω which forms a doublet under S-duality with the T-dual $\tilde{\omega}$ of the component of the IIA vector field along the T-duality direction. These two scalars are necessary in order to compensate for the two degrees of freedom associated to the two transverse scalars that have been eliminated from the action through the gauging procedure. This scalar doublet does however not play a role in our construction and has therefore been set to zero in our action above. The action (2.3) should then be regarded as a truncated action suitable for the study of the wrapped monopole in the $AdS_5 \times Y_5$ background.

In the Chern-Simons part of the action we find a coupling to $N^{(7)}$, the tensor field dual to the Taub-NUT Killing vector ℓ_μ , considered as a 1-form. The contraction $i_\ell N^{(7)}$ is the field to which a KK-monopole with Taub-NUT direction ℓ^μ couples minimally (see [18]). In (2.3) this field is further contracted with the second isometric direction k^μ , indicating that the monopole is wrapped along this direction. More importantly for our construction below, the second coupling in the CS action involves the momentum operator $P[k^{(1)}/k^2]$, associated to the isometric direction with Killing vector k^μ . Therefore, it is possible to induce momentum charge in this isometric direction, with an appropriate choice of \mathcal{F} . As we will show below, we will make use of this coupling to let the monopole propagate along the isometric direction ψ . Finally, the dots indicate couplings to other Type IIB background fields which do not play a role in our construction.

2.3 The giant graviton solution

Let us now particularise the action (2.3) to our probe KK-monopole. We take our monopole wrapped on the transverse Y_5 . Therefore the fibre direction of the decomposition of Y_5 as a $U(1)$ fibre bundle over M_4 is identified as the isometric worldvolume direction in (2.3), and M_4 as the effective four-dimensional spatial worldvolume. The Taub-NUT direction is taken along the S^1 fibre direction of the S^3 contained in AdS_5 . Therefore, we have explicitly

$$k^\mu = \delta_\psi^\mu, \quad \ell^\mu = \delta_\chi^\mu. \tag{2.7}$$

With this choice of Killing directions the contribution of the 4-form RR-potential of the $AdS_5 \times Y_5$ background to the action vanishes. This is so because both couplings to $C^{(4)}$, in (2.6) and in (2.3), involve directions along Y_5 , plus the Taub-NUT direction, χ , which lives in the AdS_5 part of the spacetime.

Furthermore, in order to induce momentum charge in the ψ direction we choose the worldvolume vector field V proportional to the curvature tensor of the Y_5 fibre connection B , such that

$$\mathcal{F} = * \mathcal{F}, \quad \int_M \mathcal{F} \wedge \mathcal{F} = 2n^2 \Omega_M, \tag{2.8}$$

where Ω_M is the volume of M_4 and the Hodge star is taken with respect to the metric on this manifold³. With this Ansatz \mathcal{F} satisfies trivially the Bianchi identities. Then, through

³The integral above is non-zero because it is the product of two integrals, $\oint \mathcal{F}$, over non-trivial two-cycles in M_4 (see for example [19]). Since $\oint \mathcal{F} = 2\pi n$ due to Dirac quantisation condition, n represents the

the second coupling in the Chern-Simons part of the action (2.3), we have that

$$\frac{T_4}{2} \int_{\mathbb{R} \times M_4} P[k^{-2}k^{(1)}] \wedge \mathcal{F} \wedge \mathcal{F} = n^2 T_W \int dt P[k^{-2}k^{(1)}], \quad (2.9)$$

where we have used the fact that the tension of the wrapped monopole is related to the tension of the point-like object carrying momentum charge (the gravitational wave) through $\Omega_M T_4 = T_W$. Therefore, with this Ansatz for \mathcal{F} , we are dissolving in the worldvolume n^2 momentum charges in the ψ direction. Notice that the instantonic nature of (2.8) guarantees that the equations of motion for \mathcal{F} are satisfied.

A second remarkable property of the Ansatz (2.8) is that the determinant of $(P[g]_{ab} + \mathcal{F}_{ab})$ is a perfect square [9, 11], such that the Born-Infeld part of the action (2.3) gives rise to

$$S = -T_4 \int dt d\Omega_M \frac{Lr^2}{4} \sqrt{\left(1 + \frac{r^2}{L^2}\right) \left[\frac{L^4}{8} + \frac{4n^2}{L^2 r^2}\right]^2 |g_M|}, \quad (2.10)$$

which after integration over M_4 gives rise to the following Hamiltonian

$$H = \frac{n^2 T_W}{L} \sqrt{1 + \frac{r^2}{L^2}} \left[1 + \frac{L^6 r^2}{32n^2}\right]. \quad (2.11)$$

The energy of the configuration is therefore a function of the radial coordinate r of AdS_5 and is clearly minimised when $r = 0$, that is, when the monopole is sitting at the centre of AdS_5 . Moreover, for this value of r the energy is given by

$$E = \frac{n^2 T_0}{L} = \frac{P_\psi}{L}. \quad (2.12)$$

Therefore, the configuration that we have proposed behaves as a giant graviton: it has the energy of a massless particle with momentum P_ψ but clearly has some finite radius L , as it is wrapped around the entire Y_5 . Since it saturates a BPS bound it is a solution of the equations of motion. Finally, we should note that there is no dynamical equilibrium between the brane tension and the angular momentum, as in the traditional giant graviton configurations of [1–4]. However the stability of the configuration is still guaranteed due to the fact that it wraps the entire transverse space.

3. A microscopical description in terms of dielectric gravitational waves

It is by now well-known that the traditional giant graviton configurations of [1–4] can be described microscopically in terms of multiple gravitational waves expanding into (a fuzzy version of) the corresponding spherical brane by Myers’ dielectric effect [20]. In particular, the M5-brane giant graviton configurations of the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ spacetimes have been described in terms of multiple M-waves expanding into a fuzzy 5-sphere that is defined as an S^1 bundle over a fuzzy CP^2 [9]. A non-trivial check of the validity of this

winding number of D3-branes wrapped around each of the two cycles. For our construction we have chosen the same winding number in both cycles in order to preserve the self-duality condition (2.8).

description is that it agrees exactly with the spherical brane description in [1, 2] when the number of gravitons is very large.

In this spirit one would expect that the KK-monopole giant graviton configuration constructed in the previous section would be described microscopically in terms of dielectric Type IIB gravitational waves expanding into a fuzzy Y_5 . The fuzzy version of general quasi-regular Sasaki-Einstein manifolds is however not known. Therefore we will restrict to the case in which Y_5 coincides with the 5-sphere. In this case we will see that the waves expand into a fuzzy 5-sphere, defined as an S^1 bundle over a fuzzy CP^2 , i.e. the same type of fuzzy manifolds involved in the description used in [9] (see also [10, 11]).

Consider now a number N of coinciding gravitational waves in the $AdS_5 \times S^5$ background. The action describing multiple gravitational waves in Type IIB was constructed in [21, 22] and it contains the following couplings:

$$S_W = -T_W \int d\tau \text{STr} \left\{ k^{-1} \sqrt{-P \left[E_{\mu\nu} - E_{\mu i} (Q^{-1} - \delta)^i_j E^{jk} E_{k\nu} \right] \det Q} \right\} \quad (3.1)$$

$$+ T_W \int d\tau \text{STr} \left\{ -P[k^{-2}k^{(1)}] - iP[(i_X i_X) i_\ell C^{(4)}] - \frac{1}{2} P[(i_X i_X)^2 i_\ell i_k N^{(7)}] + \dots \right\}$$

where

$$E_{\mu\nu} = \mathcal{G}_{\mu\nu} - k^{-1} \ell^{-1} (i_k i_\ell C^{(4)})_{\mu\nu}, \quad \mathcal{G}_{\mu\nu} = g_{\mu\nu} - k^{-2} k_\mu k_\nu - \ell^{-2} \ell_\mu \ell_\nu,$$

$$Q^\mu{}_\nu = \delta^\mu_\nu + ik\ell [X^\mu, X^\rho] E_{\rho\nu}, \quad ((i_X i_X) i_\ell C_4)_\lambda = [X^\rho, X^\nu] \ell^\mu C_{\mu\nu\rho\lambda}. \quad (3.2)$$

$$(i_\ell i_k \Omega)_{\mu_1 \dots \mu_n} = \ell^\rho k^\nu \Omega_{\nu\rho\mu_1 \dots \mu_n},$$

This action is valid to describe waves propagating in backgrounds which contain two isometric directions, parametrised in the action by the Killing vectors k^μ and ℓ^μ . The wave action is actually a gauged sigma model in which the embedding scalars associated to the Killing directions are projected out. The physical meaning of k^μ is that it corresponds to the propagation direction of the gravitational waves, while ℓ^μ is an isometry direction inherited from the T-duality operation involved in the construction of the action (see [21] and [22] for more details).

Although the only non-zero term in the Chern-Simons action in the $AdS_5 \times S^5$ background is the dipole coupling to $C^{(4)}$, it is worth calling the attention to the quadrupole coupling to $N^{(7)}$. Indeed, this coupling shows that the waves can expand via a quadrupole effect into a monopole with Taub-NUT direction parametrised by l^μ and further wrapped around the k^μ direction. This monopole will then act as the source of $i_\ell i_k N^{(7)}$.

Let us now use the action (3.1) to describe microscopically the KK-monopole of the previous section in the $AdS_5 \times S^5$ background. In this case $M_4 = CP^2$ and ds_M^2 stands for the Fubini-Study metric on the CP^2 (see for instance [23]). The identification of the isometry directions of (3.1) is then obvious. In order to account for the momentum in the worldvolume of the monopole, we identify the propagation direction of the waves with the S^5 fibre direction ψ , while the extra isometry will be identified with the Taub-NUT direction χ of the monopole:

$$k^\mu = \delta^\mu_\psi, \quad \ell^\mu = \delta^\mu_\chi. \quad (3.3)$$

With this choice of Killing vectors it is clear that the contribution of $C^{(4)}$ to the action vanishes, both in the BI and in the CS parts. Therefore, any dielectric effect will be purely gravitational [24–26].

Furthermore, we will have the gravitational waves expand into the entire five-sphere, whose fuzzy version we choose to describe as an S^1 bundle over the fuzzy CP^2 . Therefore, we take the non-commutative scalars in (3.1) to parametrise the fuzzy CP^2 base of the S^5 .

The fuzzy CP^2 has been extensively studied in the literature. For its use in the giant graviton context we refer to [9, 10], where more details on the construction that we sketch below can be found. CP^2 is the coset manifold $SU(3)/U(2)$ and can be defined as the submanifold of \mathbb{R}^8 determined by the constraints

$$\sum_{i=1}^8 x^i x^i = 1, \quad \sum_{j,k=1}^8 d^{ijk} x^j x^k = \frac{1}{\sqrt{3}} x^i, \quad (3.4)$$

where d^{ijk} are the components of the totally symmetric $SU(3)$ -invariant tensor. A fuzzy version of CP^2 can then be obtained by imposing the conditions (3.4) at the level of matrices (see for example [27]). We define a set of coordinates X^i ($i = 1, \dots, 8$) as

$$X^i = \frac{T^i}{\sqrt{(2N-2)/3}}, \quad (3.5)$$

where T^i are the $SU(3)$ generators in the N -dimensional irreducible representations $(k, 0)$ or $(0, k)$, with $N = (k+1)(k+2)/2$. The first constraint in (3.4) is then trivially satisfied through the quadratic Casimir $(2N-2)/3$ of the group, whereas the rest of the constraints are satisfied for any N (see [27, 9] for the details). The commutation relations between the X^i are given by

$$[X^i, X^j] = \frac{i f^{ijk}}{\sqrt{(2N-2)/3}} X^k, \quad (3.6)$$

with f^{ijk} the structure constants of $SU(3)$ in the algebra of the Gell-Mann matrices $[\lambda^i, \lambda^j] = 2i f^{ijk} \lambda^k$.

Substituting the Ansätze (3.5) and (3.3) in the action (3.1), we find

$$S = -T_W \int d\tau \text{STr} \left\{ L^{-1} \sqrt{\left(1 + \frac{r^2}{L^2}\right) \left[1 + \frac{3L^6 r^2}{32(N-1)} X^2\right]^2} \right\}, \quad (3.7)$$

up to order N^{-2} . Here we have dropped those contributions to $\det Q$ that vanish when taking the symmetrised trace, and ignored higher powers of N which will vanish in the large N limit.⁴

Taking the symmetrised trace we arrive at the following Hamiltonian

$$H = \frac{NT_W}{L} \sqrt{1 + \frac{r^2}{L^2}} \left[1 + \frac{L^6 r^2}{32(N-1)}\right], \quad (3.8)$$

⁴These terms cannot be nicely arranged into higher powers of the quadratic Casimir without explicit use of the constraints.

which, in the large N limit, is in perfect agreement with the Hamiltonian for the spherical KK-monopole, given by (2.11). To see this we should recall that in the macroscopical description the momentum charge of the configuration is the number of waves dissolved in the worldvolume, and is therefore given by n^2 . In the microscopical description the momentum charge is given directly by the number of microscopic waves, N . Therefore in the large N limit N and n^2 must coincide. The Hamiltonian in (3.8) is also a function of the radial coordinate r of AdS_5 and it is minimised at $r = 0$ where it takes the value $E = P_\psi/L$, thus corresponding to a giant graviton configuration.

4. A possible interpretation in the dual field theory

In this section we try to give a possible field theoretic interpretation of the giant graviton configuration that we have studied, along the lines of [3] (see also [28–31]). We will discuss the giant graviton configuration in the $AdS_5 \times S^5$ background, but we will later speculate on a possible generalisation to other Sasaki-Einstein spaces.

We have learnt in the previous sections that the fibre direction in the S^3 contained in AdS_5 plays a crucial role in the construction of the giant graviton configuration, as it is identified with the Taub-NUT direction of the monopole. Therefore, it is useful to work in the global patch for AdS.

It is then natural to consider the dual field theory as living in $\mathbb{R} \times S^3$, where there is a conformal coupling to the curvature. The bosonic piece of the action reads

$$S = \frac{1}{2} \int dt d\Omega_3 \text{Tr} \left\{ \partial_\mu \Phi_a^* \partial^\mu \Phi_a + \frac{1}{L^2} \Phi_a^* \Phi_a + \frac{1}{4g^2} [\Phi_a, \Phi_b^*]^2 \right\}, \quad (4.1)$$

where L is the radius of the S^3 and the Φ_a (with $a = 1, 2, 3$) are the complexification of the 6 adjoint real scalars X^i of $\mathcal{N} = 4$ SYM. After defining $\Phi_a = X^a + iX^{a+3}$, only an $SU(3)$ subgroup of the original $SO(6)$ R-symmetry group remains explicit.

Regarding S^3 as an S^1 bundle over S^2 it seems a consistent truncation to assume that the Φ_a do not depend on the fibre coordinate. Actually, this will be the field theory analogue of the fact that this direction corresponds to the Taub-NUT direction of the monopole on the gravity side. Taking adapted coordinates to the $U(1)$ fibration we have

$$S = 2\pi \int dt \int d\Omega_2 \text{Tr} \left\{ -\partial_t \Phi_a^* \partial_t \Phi_a - 4\Phi_a^* \Delta_{S^2} \Phi_a + \frac{1}{L^2} \Phi_a^* \Phi_a + \frac{1}{4g^2} [\Phi_a, \Phi_b^*]^2 \right\}, \quad (4.2)$$

where Δ_{S^2} is the Laplacian in the 2-sphere.

We can then expand the scalars in spherical harmonics $\Phi_a^{(lm)}$ on the two-sphere. Given that we will be interested in the lowest energy modes, we will truncate all of them except the massless mode $\Phi_a^{(0)}$, which corresponds to the constant mode on the S^2 . Furthermore, we consider the following Ansatz for the gauge and $SU(3)$ dependence of our fields

$$\Phi_a^{(0)} = e^{if(t)} \mathcal{M}_a \otimes J_a, \quad (4.3)$$

where $f(t)$ is an arbitrary function of time, the traceless matrix \mathcal{M}_a is defined as

$$\mathcal{M}_a = \text{diag} \left(v_a, -\frac{v_a}{M-1}, \dots, -\frac{v_a}{M-1} \right), \quad (4.4)$$

and the J_a 's are SU(2)-generators in a k -dimensional representation. Since the total rank of the gauge group is N we should have that $kM = N$. Indeed, in this branch the gauge group breaks into SU(M). The gauge transformations which are left are those of the form

$$\Phi_a \rightarrow g\Phi_a g^\dagger, \quad g = \text{diag}(\tilde{g}_k, \dots, \tilde{g}_k), \quad (4.5)$$

where the \tilde{g}_{k_i} are SU(2) gauge transformations of dimension k .

The action then reduces to

$$S = 8\pi^2 \frac{C_2(k)M}{M-1} \int dt \left[-(f')^2 v^2 + \frac{v^2}{L^2} \right], \quad (4.6)$$

where $v^2 = \delta^{ab} v_a v_b$ is to be interpreted as a non-dynamical parameter whose value will determine the minima of the potential. In addition, $C_2(k)$ is the Casimir of the SU(2) k -dimensional representation. In this action $f(t)$ is a cyclic variable and therefore its conjugate momentum, p , will be conserved. The Hamiltonian is given by

$$H = p^2 \frac{(M-1)}{32\pi^2 v^2 C_2(k)M} + \frac{8\pi^2 M C_2(k) v^2}{(M-1)L^2}, \quad (4.7)$$

which has a minimum for $v^2 = \frac{(M-1)}{32\pi^2 C_2(k)M} pL$. Remarkably the on-shell energy is precisely the dispersion relation

$$E = \frac{p}{L}. \quad (4.8)$$

Therefore, the configuration (4.3) can be seen as a massless particle. Furthermore, out of the original SU(3) rotating our Φ_a just an SU(2) survives, given that with our Ansatz the Φ_a become a vector of SU(2). Thus, the moduli space reduces to SU(3)/U(2), which is precisely the symmetry of CP^2 as a coset space, which is in turn the manifold wrapped by our KK-monopole.

Given that our construction of the wrapped KK-monopole works not just in the S^5 case, but also in more generic spaces, we expect a similar field theory description for the dual of a generic Sasaki-Einstein space. In supporting this claim, let us notice that the potential term did not play any role in the S^5 case, because with the Ansatz we assumed, it vanishes. In the generic case, we will assume a similar Ansatz for our fields, namely

$$X_\alpha = e^{if(t)} \mathcal{M} \otimes G_\alpha, \quad (4.9)$$

where now for simplicity we take the same \mathcal{M} matrix as before but with all the v 's identical. The G_α are the generators of the global symmetry group G . Given this Ansatz, we expect that the superpotential does not play any role, not even in the most generic $Y^{p,q}$ case. In addition, since we take AdS in the global patch, the field theory will be defined in $\mathbb{R} \times S^3$, so we will always have the conformal coupling to the curvature. Just this term, together with the kinetic energy, is enough to reproduce a dispersion relation of the form $E \sim p$.

In the general case we can also regard the S^3 as an S^1 bundle over S^2 , and take our fields independent of the U(1). This is the field theory counterpart of the presence of the Taub-NUT direction in the gravity side. In addition, out of the full global symmetry group

G we will just keep the subgroup g compatible with our Ansatz,⁵ so we would expect a moduli space of the form G/g . Let us consider for example the conifold. In this case the global symmetry group is $SU(2) \times SU(2)$. Therefore we have to first reduce it to the diagonal $SU(2)$ and then take the conifold scalars A and B to be $e^{if(t)}\mathcal{M}$. This leaves an $[SU(2) \times SU(2)]_D/U(1)$ moduli space, which is the symmetry of 2 2-spheres, and this is in turn what one would get from the gravity side.

Finally, we would like to note that our construction is quite generic. From the gravity point of view we just require that the momentum of the KK-monopole wrapping the five-dimensional space is taken along a $U(1)$ fibre direction, and that its Taub-NUT direction is along the S^1 in the decomposition of the $S^3 \subset AdS_5$ as an S^1 bundle over S^2 . In the field theory side our requirements are also quite generic. The existence of the Taub-NUT direction is reflected on the fact that the SCFT is defined in $\mathbb{R} \times S^3$ and S^3 is taken as S^1 over S^2 . In addition, our description is not sensitive to the superpotential, which we believe is the counterpart to the fact that the KK-monopole wraps the whole 5-dimensional manifold. Then, we are left with the kinetic term and the conformal coupling to curvature, which is enough to ensure the right dispersion relation. Since in general our Ansatz will reduce the original global symmetry, the moduli space will be G/g , which we believe will correspond in general to the symmetry of the 4-dimensional base which the KK-monopole wraps.

5. Conclusions

In this letter we have constructed a new type of giant graviton solution in $AdS_5 \times Y_5$, with Y_5 a quasi-regular Sasaki-Einstein manifold. This solution consists on a Kaluza-Klein monopole with internal momentum, wrapped around the entire Y_5 and with Taub-NUT direction along the AdS_5 part.

Although the dynamics of this monopole can be described using the effective action for the Type IIB Kaluza-Klein monopole constructed in [13], this action is only known to quadratic order in the self-dual 2-form of its six-dimensional (2,0) tensor multiplet field content. However, given that Y_5 can be decomposed as a $U(1)$ bundle over a four-dimensional Kähler-Einstein manifold M_4 , it is possible to use the action for a monopole wrapped on a $U(1)$ direction to describe it. This action, having the field content of the five-dimensional (1,1) vector multiplet, is known to all orders. Moreover, it is possible to induce momentum charge along the $U(1)$ direction through a suitably chosen worldvolume vector field with non-zero instanton number. Using the action for a $U(1)$ wrapped monopole we have shown that the energy of the configuration depends on its radial position in the AdS space and behaves as a massless particle when put in the origin, while having the size of the Y_5 .

Given that the spherical monopole carries a non-vanishing momentum charge there should be a microscopical description of the same configuration in terms of expanding gravitational waves. This description would involve however the fuzzy version of Y_5 , which

⁵Note that g may involve discrete subgroups such as \mathbb{Z}_k

is not known in general. Therefore, we have restricted to the case in which $Y_5 = S^5$, and let the multiple dielectric gravitational waves expand into a fuzzy 5-sphere. The fuzzy 5-sphere built up by the gravitational waves is constructed as an Abelian fibre bundle over a fuzzy CP^2 , a construction that has been used before in the study of the traditional giant gravitons and the baryon vertex with magnetic flux. The configuration thus obtained turns out to exactly agree in the limit of large number of waves with the effective KK-monopole description.

We believe there are several reasons why this new giant graviton solution has not been found earlier in the literature. First of all, since it has no relation with the stringy exclusion principle it is not straightforward to find the corresponding state in the CFT side. Moreover, as we have shown in section 4, our scalar field configuration breaks the R-symmetry group in a rather peculiar way, making explicit the U(1) fibre structure of the S^3 . Secondly, the fact that it is built up from a Kaluza-Klein monopole and not from a more ordinary type of brane, makes our construction more involved.

An interesting question to answer would be whether the KK-monopole giant graviton solution is supersymmetric or not. This is however difficult to answer, on the one hand due to the form of the Killing spinors in the particular coordinate system that we are using and, on the other hand, due to the fact that the kappa-symmetry for the Kaluza-Klein monopole is not known. Yet, the fact that the configuration is massless implies that it saturates a BPS bound, which hints to the fact that it probably preserves some fraction of the supersymmetry. We would like to leave this problem for future investigations.

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